Inference at $*$ 1 1 0
of proof for Lemma neg_mul_arg_bounds:

1. $a : \mathbb{Z}$
2. $b : \mathbb{Z}$
3. $((\text{-}(a * b)) > (\text{-}0)) \iff ((((\text{-}a) > (\text{-}0)) \,\&\, (b > 0)) \vee (((\text{-}a) < (\text{-}0)) \,\&\, (b < 0)))$

$\vdash ((a * b) < 0) \iff (((a < 0) \,\&\, (b > 0)) \vee ((a > 0) \,\&\, (b < 0)))$

by PERMUTE{1:n,
2:n,
3:n,
4:n,
5:n,
6:n,
7:n,
8:n,
9:n,
10:n,
11:n,
12:n,
13:n,
13:n,
5:n,
14:n,
15:n,
16:n,
17:n,
18:n,
18:n,
14:n,
5:n,
19:n,
20:n}

1: . . . . .wf. . . . . NILNIL

$\vdash ((\text{-}(a * b)) > (\text{-}0)) \in \mathbb{P}_1$

2: . . . . .wf. . . . . NILNIL

$\vdash (0 > (a * b)) \in \mathbb{P}_1$

3: . . . . .wf. . . . . NILNIL

$\vdash ((((\text{-}a) > (\text{-}0)) \,\&\, (b > 0)) \vee (((\text{-}a) < (\text{-}0)) \,\&\, (b < 0))) \in \mathbb{P}_1$

4: . . . . .wf. . . . . NILNIL

$\vdash (((0 > a)\ \&\ (b > 0)) \vee ((a > 0)\ \&\ (b < 0))) \in \mathbb{P}_1$
5: . . . . .wf. . . . . NILNIL

$\vdash 0 \in \mathbb{Z}$
6: . . . . .wf. . . . . NILNIL

$\vdash (a * b) \in \mathbb{Z}$
7: . . . . .wf. . . . . NILNIL

$\vdash (((\text{-}a) > (\text{-}0))\ \&\ (b > 0)) \in \mathbb{P}_1$
8: . . . . .wf. . . . . NILNIL

$\vdash ((0 > a)\ \&\ (b > 0)) \in \mathbb{P}_1$
9: . . . . .wf. . . . . NILNIL

$\vdash (((\text{-}a) < (\text{-}0))\ \&\ (b < 0)) \in \mathbb{P}_1$
10: . . . . .wf. . . . . NILNIL

$\vdash ((a > 0)\ \&\ (b < 0)) \in \mathbb{P}_1$
11: . . . . .wf. . . . . NILNIL

$\vdash ((\text{-}a) > (\text{-}0)) \in \mathbb{P}_1$
12: . . . . .wf. . . . . NILNIL

$\vdash (0 > a) \in \mathbb{P}_1$
13: . . . . .wf. . . . . NILNIL

$\vdash (b > 0) \in \mathbb{P}_1$
14: . . . . .wf. . . . . NILNIL

$\vdash a \in \mathbb{Z}$
15: . . . . .wf. . . . . NILNIL

$\vdash (b > 0) = (b > 0)$
16: . . . . .wf. . . . . NILNIL

$\vdash ((\text{-}a) < (\text{-}0)) \in \mathbb{P}_1$
17: . . . . .wf. . . . . NILNIL

$\vdash (a > 0) \in \mathbb{P}_1$
18: . . . . .wf. . . . . NILNIL

$\vdash (b < 0) \in \mathbb{P}_1$
19: . . . . .wf. . . . . NILNIL

$\vdash (b < 0) = (b < 0)$

20:

3. $(0 > (a * b)) \iff (((0 > a)\ \&\ (b > 0)) \vee ((a > 0)\ \&\ (b < 0)))$
$\vdash ((a * b) < 0) \iff (((a < 0)\ \&\ (b > 0)) \vee ((a > 0)\ \&\ (b < 0)))$

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http://www.nuprl.org/FDLcontent/p0_942988_/p121_74264_{neg_mul_arg_bounds}1_1_0.html